How to Cast a Ray From the Eye Through a Given Pixel in the Screen

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We begin with a pixel $P = \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$ in device coordinates (an x,y pair between 0,0 and n_x, n_y, where n_x is the width, in pixels of the screen) and a camera: an eye position, a lookAt target position, an up vector, and either an fov or a distance (used to determine distance between the eye and the screen).

We seek to create a ray with its origin at the eye and its direction pointing towards the given pixel (converted into world co-ordinates). This ro is quite simply our eye position. rd is P_w - eye, where P_w is our pixel converted into world co-ordinates. We do this conversion as a sequence of four affine transformations to P.

Firstly, we perform a translation operation T_1, with $d = \tan(.5 * fovY)$ or the provided distance:

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{n_x}{2} \\ 0 & 1 & 0 & -\frac{n_y}{2} \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Our intermediate point $P = (x, y, d, 1)$, where x and y are between $(-n_x/2, -n_y/2)$ and $(n_x/2, n_y/2)$. Apparently to preserve aspect ratio we now have to scale P by S_2:

$$\begin{bmatrix} \frac{-h}{n_y} & 0 & 0 & 0 \\ 0 & \frac{w}{n_x} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Our coordinates are now between $(h/2, -w/2)$ and $(-h/2, w/2)$ Where h and w are the height and width of the screen in world space. We use $w = \frac{n_x}{n_y}$, $h = 2d * \tan fovY$.

At this point to convert into world coordinates from our view coordinates, we need to rotate according to our camera rotation, and then translate according to it. To do so we define:

$$w = \frac{lookAt - lookFrom}{||lookAt - lookFrom||}, u = \frac{up \times w}{||up \times w||}, v = w \times u \tag{3}$$

$$R_3 = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_4 = \begin{bmatrix} 1 & 0 & 0 & lookFrom_x \\ 0 & 1 & 0 & lookFrom_y \\ 0 & 0 & 1 & lookFrom_x \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Thus, $P_w = T_4 \times R_3 \times S_2 \times T_1 \times P$, and as before $ro = lookFrom$, $rd = Pw - lookFrom$.